References

[1] Toepler, A., "Beobachtungen nach einer Neuen Optischen Methode," *Poggendorf's Annalen der Physik und Chemie*, Vol. 127, No. 4, May 1866, pp. 556–580.

[2] Holder, D. W. and North, R. J., "Schlieren Methods," NPL Notes on Applied Science No. 31, Her Majesty's Stationery Office, London, 1963.

[3] Edmondson, R. B., Gayhart, E. L., and Olsen, H. L., "Radial Symmetry in a Schlieren Image," *Journal of the Optical Society of America*, Vol. 42, No. 12, Dec. 1952, p. 984.

[4] Barry, F. W. and Edelman, G. M., "An Improved Schlieren Apparatus," *Journal of the Aeronautical Sciences*, Vol. 15, No. 6, June 1948, p. 364.

[5] Prescott, R. and Gayhart, E. L., "A Method of Correction of Astigmatism in Schlieren Systems," *Journal of the Aeronautical Sciences*, Vol. 18, No. 1, Jan. 1951, p. 69.

[6] Cords, P. H., Jr., "A High Resolution, High Sensitivity Color Schlieren Method," *Society of Photo-Optical Instrumentation Engineers Journal*, Vol. 6, No. 3, Feb.-March 1968, pp. 85–88.

A Modified Form of the Coles Compressibility Transformation

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Nomenclature

$c_f, \bar{c}_f$ = local skin-friction coefficients
F = nondimensional transpiration rate $-\rho_w v_w/\rho_e u_e$
I = integral defined by [Eq. (7)]
$R_y, R_{\bar{y}}$ = Reynolds number based on normal coordinates
$R_\delta, R_{\bar{\delta}}$ = Reynolds number based on boundary-layer thicknesses
$R_\theta, R_{\bar{\theta}}$ = Reynolds number based on momentum thicknesses
$u, \bar{u}$ = streamwise velocity components
$v, \bar{v}$ = normal velocity components
$x, \bar{x}$ = streamwise coordinates
$y, \bar{y}$ = normal coordinates
α = $R_{\bar{y}}/R_{\bar{\delta}}$
$\eta, \varkappa, \xi, \sigma$ = scaling parameters of the transformation
$\tau, \bar{\tau}$ = shear stresses
$\mu, \bar{\mu}$ = coefficients of laminar viscosity
Π = Coles wake parameter
$\rho, \bar{\rho}$ = densities
$\tilde{\sigma}$ = $\sigma\mu_e/\bar{\mu}$
$\psi, \bar{\psi}$ = stream functions:
$$\begin{cases} \partial\psi/\partial y = \rho u; \ \partial\psi/\partial x = \rho_w v_w - \rho v \\ \partial\bar{\psi}/\partial\bar{y} = \bar{\rho}\bar{u}; \ \partial\bar{\psi}/\partial\bar{x} = \bar{\rho}\bar{v}_w - \bar{\rho}\bar{v} \end{cases}$$

Subscripts

e = conditions external to boundary layer
w = conditions at the wall
( ) = variables of the *VP* flow
($\bar{}$) = variables of the *CP* flow
($\tilde{}$) = normalization with respect to corresponding external value; e.g., $\tilde{u} = u/u_e$

Introduction

NUMEROUS investigators[1,2] have analyzed velocity profiles obtained in high-speed flows over impermeable flat plates by means of the Coles compressibility transformation.[3] In all cases it was found that the velocity defect portion of the transformed profiles was not well correlated in the sense that the flat plate value of the Coles wake parameter Π was not recovered. A similar result was also observed for the case of low-speed flow with foreign gas injection.[4] These effects are demonstrated by some representative results presented in Fig. 1.

It has been suggested[5,6] that this behavior may be caused by the use of a Howarth-Dorodnytsin (HD) scaling throughout the boundary layer and that such use may not be appropriate in the wake region. The purpose of this Note is to outline a modification of the original transformation which permits suppression of the HD scaling, while retaining certain other useful features of the transformation; e.g., detailed mapping of the "law of the wall" region.

Modified Transformation

The modification introduces a new parameter $\varkappa(y)$, such that the y stretching takes the form

$$(\bar{\rho}/\rho)(\partial\bar{y}/\partial y)_x = \eta(x)\varkappa(y) \tag{1}$$

In order to preserve the original correspondence between the axial velocity components given by $u/u_e = \bar{u}/\bar{u}_e \equiv \tilde{u}$, it is also necessary to modify the corresponding stretching for the stream function according to†

$$(\partial\bar{\psi}/\partial\psi)_x = \sigma(x)\varkappa(y) \tag{2}$$

The development from this point follows that of Coles.[3] In particular, with the axial coordinates related by $d\bar{x}/dx = \xi(x)$, the correspondence between velocity components becomes‡

$$u = (\eta/\sigma)\bar{u} \tag{3}$$

$$\rho v = (1/\sigma\varkappa)[\bar{\rho}\bar{v}\xi - \bar{\rho}\bar{u}(\partial\bar{y}/\partial x)_y + (\partial\bar{\psi}/\partial x)_\psi] \tag{4}$$

while the convective term appearing in the x-wise momentum equation transforms according to

$$\rho u\partial u/\partial x + \rho v\partial u/\partial y = (\bar{\rho}\bar{u}\partial\bar{u}/\partial\bar{x} + \bar{\rho}\bar{v}\partial\bar{u}/\partial\bar{y}) \times (\rho\xi\eta^2/\bar{\rho}\sigma^2) + (\rho\eta^2/\bar{\rho}\sigma^2)(\partial\bar{u}/\partial\bar{y})(\partial\bar{\psi}/\partial x)_\psi \tag{5}$$

where, by virtue of Eq. (3), we have taken $\eta/\sigma = u_e/\bar{u}_e =$ const. Evidently, the first of these terms provides the desired transport term corresponding to the constant property (CP) flowfield, so that, in principle, transformation of the differential equations describing the variable property (VP) flow to a CP form can be completed by suitably combining the second term with the shear derivatives $\partial\tau/\partial y$ and

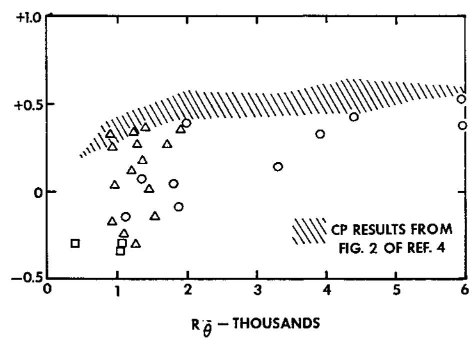

Fig. 1 Magnitude of wake component found in several VP boundary-layer configurations transformed according to the Coles stretching: triangles, low-speed helium injection results[7]; circles, adiabatic impermeable results[8]; squares, impermeable results with heat transfer.[9]

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† In view of the manner in which the stream function has been defined here, the earlier modification proposed in Ref. 4, to account properly for wall mass transfer, is included herein.

‡ The development from this point is specialized to the case of zero mass transfer and zero pressure gradient in both flows; extension to these more general cases is straightforward but is not included here for the sake of brevity.

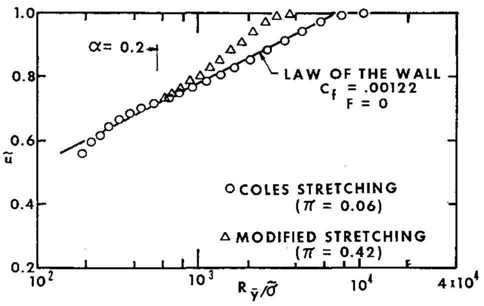

Fig. 2 Comparison of velocity profiles transformed according to the Coles stretching and the modified stretching; high-speed flow over an adiabatic, impermeable flat plate (Case 22, Ref. 8).

$\partial\bar{\tau}/\partial\bar{y}$. That is, if we let

$$(\rho\xi\eta^2/\bar{\rho}\sigma^2)\partial\bar{\tau}/\partial\bar{y} = \partial\tau/\partial y - (\rho\eta^2/\rho\sigma^2)(\partial\bar{u}/\partial\bar{y})(\partial\bar{\psi}/\partial x)_\psi \quad (6)$$

the desired form is obtained. Note, however, that this step presupposes vanishing of the second term for large y (or $\bar{y}$). To complete the formulation, the conditions which assure this behavior need to be established.

For this purpose, we begin with Eq. (2) and write for the CP stream function

$$\bar{\psi} = \sigma(x)\int_0^\psi \varkappa d\psi \quad (7)$$

Then

$$\begin{aligned}(\partial\bar{\psi}/\partial x)_\psi &= (\bar{\psi}/\sigma)(d\sigma/dx) + \left(\partial\int_0^\psi \varkappa d\psi/\partial x\right)_\psi \\ &= (\bar{\psi}/\sigma)(d\sigma/dx) - (v/u)\int_0^y \rho u(d\varkappa/dy)dy\end{aligned} \quad (8)$$

For most boundary-layer type flows, the term $\bar{\psi}\partial\bar{u}/\partial y$ can be expected to vanish at large $\bar{y}$. Accordingly, the simplest condition§ that assures the desired behavior corresponds to $d\varkappa/dy \to 0$ for large y.

In addition to transforming the partial differential equations themselves, the stretching also provides correspondences between the integral boundary-layer parameters in the two flows. For the present purpose the pertinent results include skin-friction coefficient

$$\bar{c}_f = \tilde{\rho}_w\tilde{\mu}_w\tilde{\sigma}\varkappa_w c_f \quad (9)$$

Reynolds number based on $y,\bar{y}$

$$R_{\bar{y}}/\tilde{\sigma} = \int_0^{Ry} \varkappa\tilde{\rho}dR_y \quad (10)$$

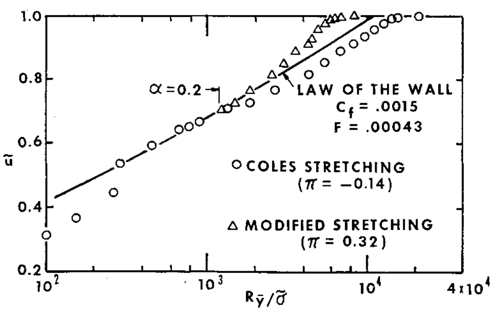

Fig. 3 Comparison of velocity profiles transformed according to the Coles stretching and the modified stretching; low-speed flow over a porous cylinder with helium injection (Run H-14, Ref. 7).

§ This is not a necessary condition, of course, but is sufficient for the present purpose; as with Coles (cf. footnote on p. 9 of Ref. 3), the possibility of a less restrictive condition is not investigated further.

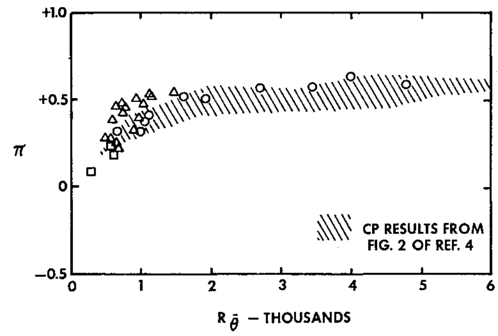

Fig. 4 Magnitude of wake component found in several VP boundary-layer configurations transformed according to the modified stretching: triangles low-speed helium injection results[7]; circles, adiabatic impermeable results[8]; squares, impermeable results with heat transfer.[9]

Reynolds number based on $\theta,\bar{\theta}$

$$R_{\bar{\theta}}/\tilde{\sigma} = R_\theta + I \quad (11)$$

where

$$I \equiv \int_0^{R\delta}(\varkappa - 1)\tilde{\rho}\tilde{u}(1-\tilde{u})dR_y \quad (12)$$

For $\varkappa \equiv 1$, of course, these correspondences reduce to the original ones obtained by Coles. Note also that for $\varkappa_w \equiv \varkappa(0) = 1$, the correspondence between skin-friction coefficients also retains its original form; however, the $R_\theta,R_{\bar{\theta}}$, correspondence does not, so that, e.g., the original "law of corresponding stations" is not recovered.

Results

In order to examine its possible usefulness, this modification has been implemented by utilizing an elementary form for the scaling parameter $\varkappa(y)$ in conjunction with a Crocco integral energy solution. In particular, we take

$$\varkappa = \begin{cases}1 \quad ; & 0 \le \alpha \le 0.2 \\ \tilde{\rho}_w/\tilde{\rho}; & 0.2 < \alpha \le 1\end{cases} \quad (13)$$

where, for numerical convenience, the scaling function $\varkappa$ has been expressed in terms of a normalized coordinate α in the CP plane. Note that this form preserves the original stretching in the vicinity of the wall and turns off the HD stretch thereafter, i.e., the y-dependent quantity $\bar{\rho}/\rho$ is replaced by the constant $\bar{\rho}/\rho_w$ in the outer region. In this connection it is emphasized here that the cutoff value $\alpha = 0.2$ is arbitrary but can be thought of as roughly corresponding to the outer edge of the law of the wall region.

This particular modification has been applied to the velocity profiles previously considered (cf., Fig. 1),¶ and typical results demonstrating the effect on the transformed profiles are presented in Figs. 2 and 3. Figure 4 summarizes all of these results and shows that the suggested modification virtually eliminates the previously observed distortion. It would appear that further exploitation of this approach is in order with particular emphasis on development of a rational and more physically meaningful basis for specification of the scaling parameter $\varkappa(y)$.

References

[1] Baronti, P. O. and Libby, P. A., "Velocity Profiles in Turbulent Compressible Boundary Layers," *AIAA Journal*, Vol. 4, No. 2, Feb. 1966, pp. 193–202.

[2] Watson, R. D. and Cary, A. M., Jr., "The Transformation of Hypersonic Turbulent Boundary Layers in Incompressible Form," *AIAA Journal*, Vol. 5, No. 6, June 1967, pp. 1202–1203.

¶ See Refs. 1 and 4 for details of the procedure used to calculate $\bar{c}_f$, Π, $\tilde{\sigma}$, etc.

[3] Coles, D. E., "The Turbulent Boundary Layer in a Compressible Fluid," R-403-PR, Sept. 1962, The Rand Corp., Santa Monica, Calif.

[4] Economos, C., "A Transformation Theory for the Compressible Turbulent Boundary Layer with Mass Transfer," *AIAA Journal*, Vol. 8, No. 4, April 1970, pp. 758–764.

[5] Morkovin, M. W., "Effects of Compressibility on Turbulent Flow," International Symposium on the Mechanique de la Turbulence, 1962, Centre National de la Recherche Scientifique, Paris, pp. 367–380.

[6] Tennekes, H., "Law of the Wall for Turbulent Boundary Layers in Compressible Flow," *AIAA Journal*, Vol. 5, No. 3, March 1967, pp. 489–492.

[7] Scott, C. T. et al., "Measurements of Velocity and Concentration Profiles for Helium Injection Into a Turbulent Boundary Layer Flowing Over an Axial Circular Cyclinder," Part I—"Experimental Results," HTL-TR-55, Feb. 1964, University of Minnesota, Minneapolis, Minn.

[8] Coles, D. E., "Measurements in the Boundary Layer on a Smooth Flat Plate in Supersonic Flow, III. Measurements in a Flat Plate Boundary Layer at the Jet Propulsion Lab.," Rept. 20-71, June 1953, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif.

[9] Winkler, E. M. and Cha, M. H., "Investigation of Flat Plate Hypersonic Turbulent Boundary Layers with Heat Transfer at a Mach Number of 5.2," Rept. 6631, Sept. 1959, U.S. Naval Ordnance Lab., White Oak, Md.

Investigation of Turbulent Recovery Factor in Hypersonic Helium Flow

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Nomenclature

M = Mach number
r = recovery factor $(T_{aw} - T_e)/(T_o - T_e)$
R = Reynolds number
T = temperature

Subscripts

e = local conditions at edge of boundary layer
aw = adiabatic wall
o = total
x = distance along plate from leading edge
v = virtual origin (at peak of recovery factor in transition region)

An accurate knowledge of the temperature recovery factor is important in the reduction of wind-tunnel heat-transfer measurements to coefficient form, especially when those measurements are made at nearly adiabatic conditions. Numerous experimental measurements of the recovery factor have been made for laminar, transitional, and turbulent boundary layers in air for Mach numbers up to 5 (i.e., Refs. 1–9). A few measurements have also been made between Mach 5 and 6.[10–12] There has actually been no pressing need for precise air recovery factor measurements at Mach numbers above 5 because the total temperatures necessary to avoid liquefaction of air at these high Mach numbers place the usual wall temperatures far below the adiabatic values. However, in helium wind tunnels, hypersonic Mach numbers can be achieved without heating the flow and therefore an accurate value of the recovery factor is again needed for reduction of heat-transfer data. The only recovery factor study previously conducted in helium involved laminar and transitional flow at Mach 6.5.[13] The present investigation was therefore initiated to experimentally obtain the turbulent recovery factor in hypersonic helium flow.

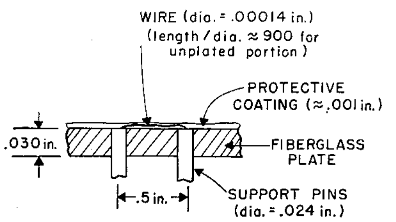

Fig. 1 Lateral cut showing instrumentation for surface temperature measurement (not to scale).

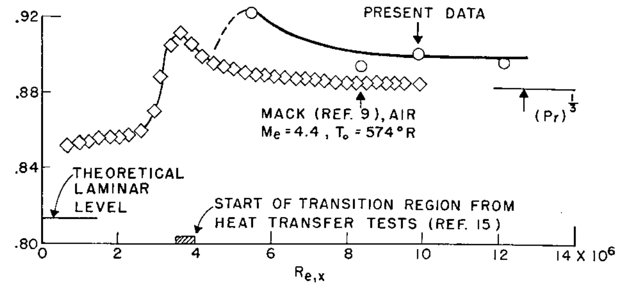

Fig. 2 Recovery factor distribution for wedge model (M_e = 6.8, helium).

The present measurements of turbulent recovery factor were made in the Langley 22-in. Mach 20 helium tunnel using a thin fiber glass plate mounted on a sharp leading edge (0.002-in. thick), 10° half-angle wedge. The local Mach number M_e on the 10° wedge was 6.8 with a negligible local pressure gradient.

The model was designed so that the fiber glass surface would be essentially insulated and thus, the equilibrium surface temperature would be very nearly the adiabatic-wall temperature. Longitudinal heat conduction was negligible because of the small cross section and low conductivity of the fiber glass plate. Calculations also indicated negligible error as the result of radiation from the tunnel wall. To accurately measure the surface temperature, the model was instrumented with thin wire (0.00014-in. diam) resistance thermometers, the ends of which were copper-plated for attachment to support pins, as shown in Fig. 1. A conservative analysis indicated that conduction through the steel support pins had only a slight (less than 0.1°R) effect on the wire temperature. Therefore, the wires were assumed to be correctly measuring the surface temperature. During the last half of the 93-sec

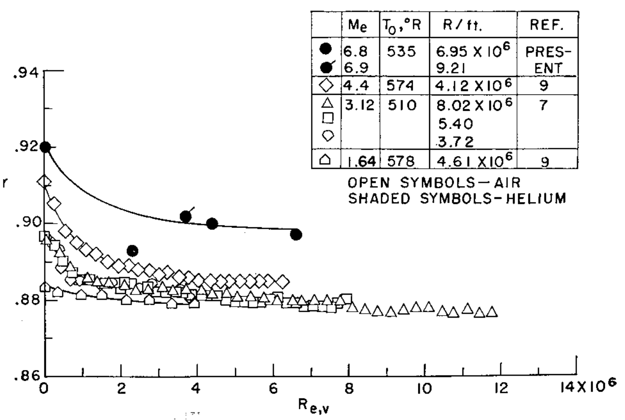

Fig. 3 Variation of turbulent recovery factor with Reynolds number based on distance from recovery factor peak in transition region.

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